Robust Separation of Reflection from Multiple Images

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Motivation

Captured Frame f = Transmitted Layer t + Reflected Layer r

The separation problem is highly ill-posed !!!

Observations & Priors

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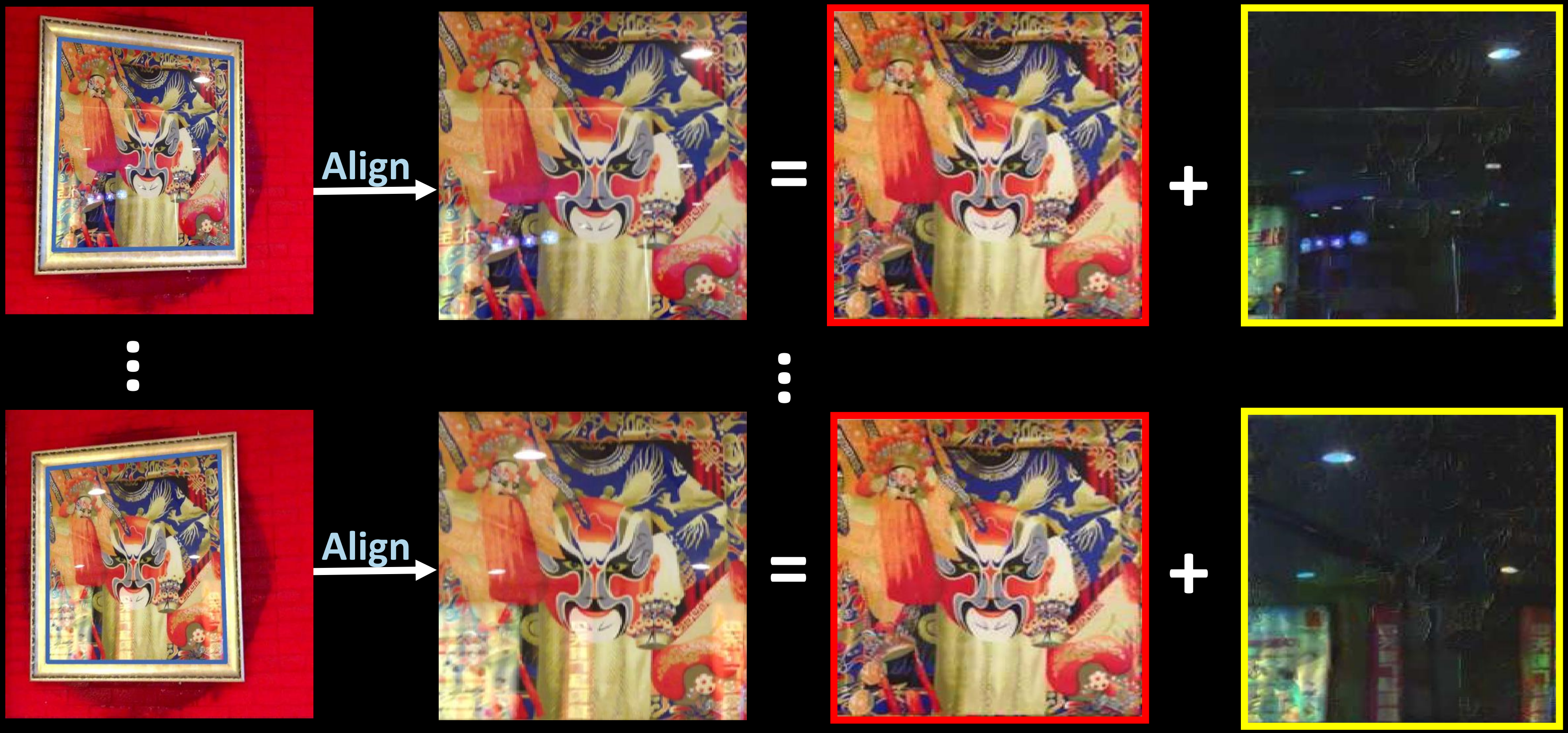
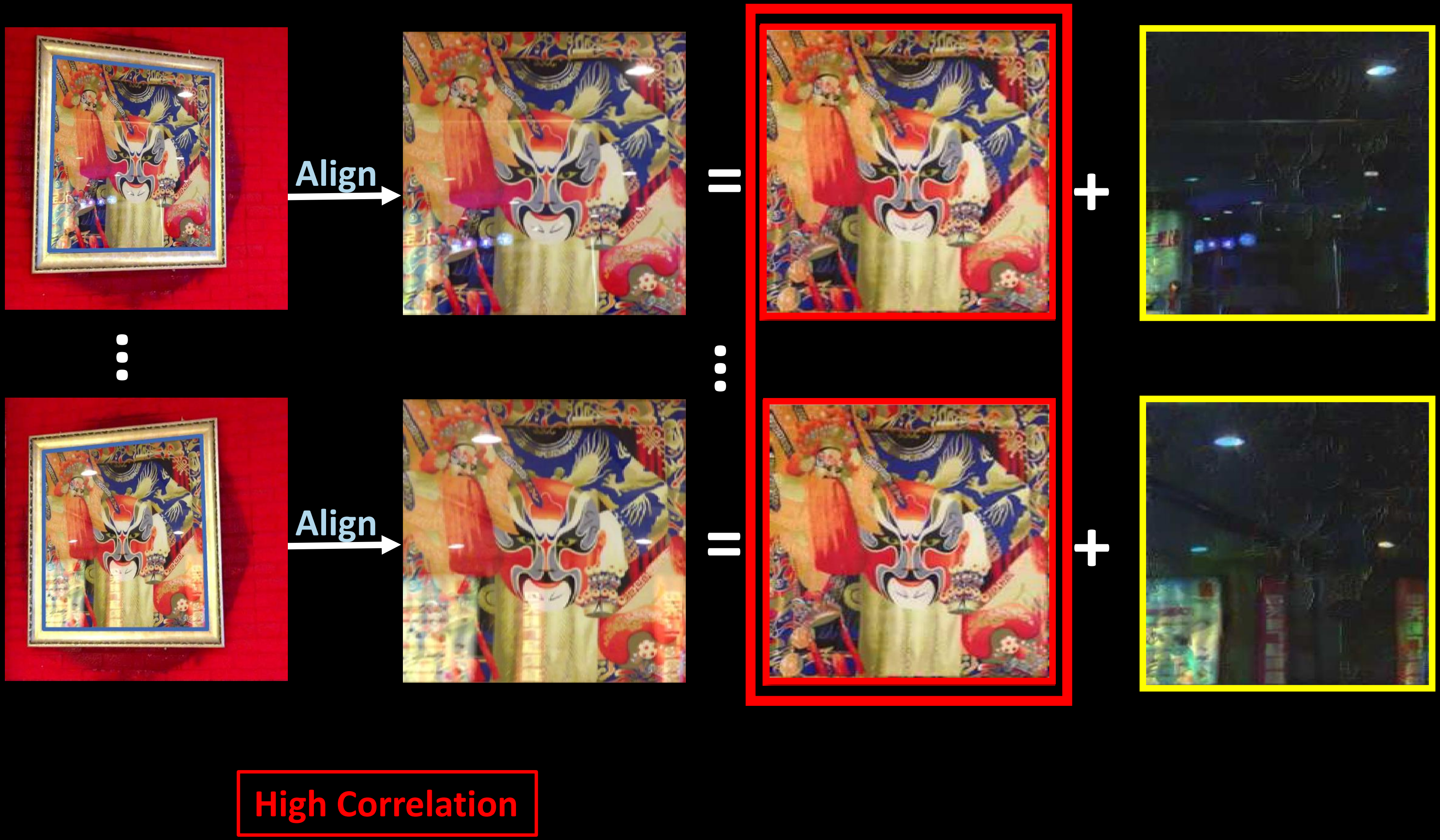
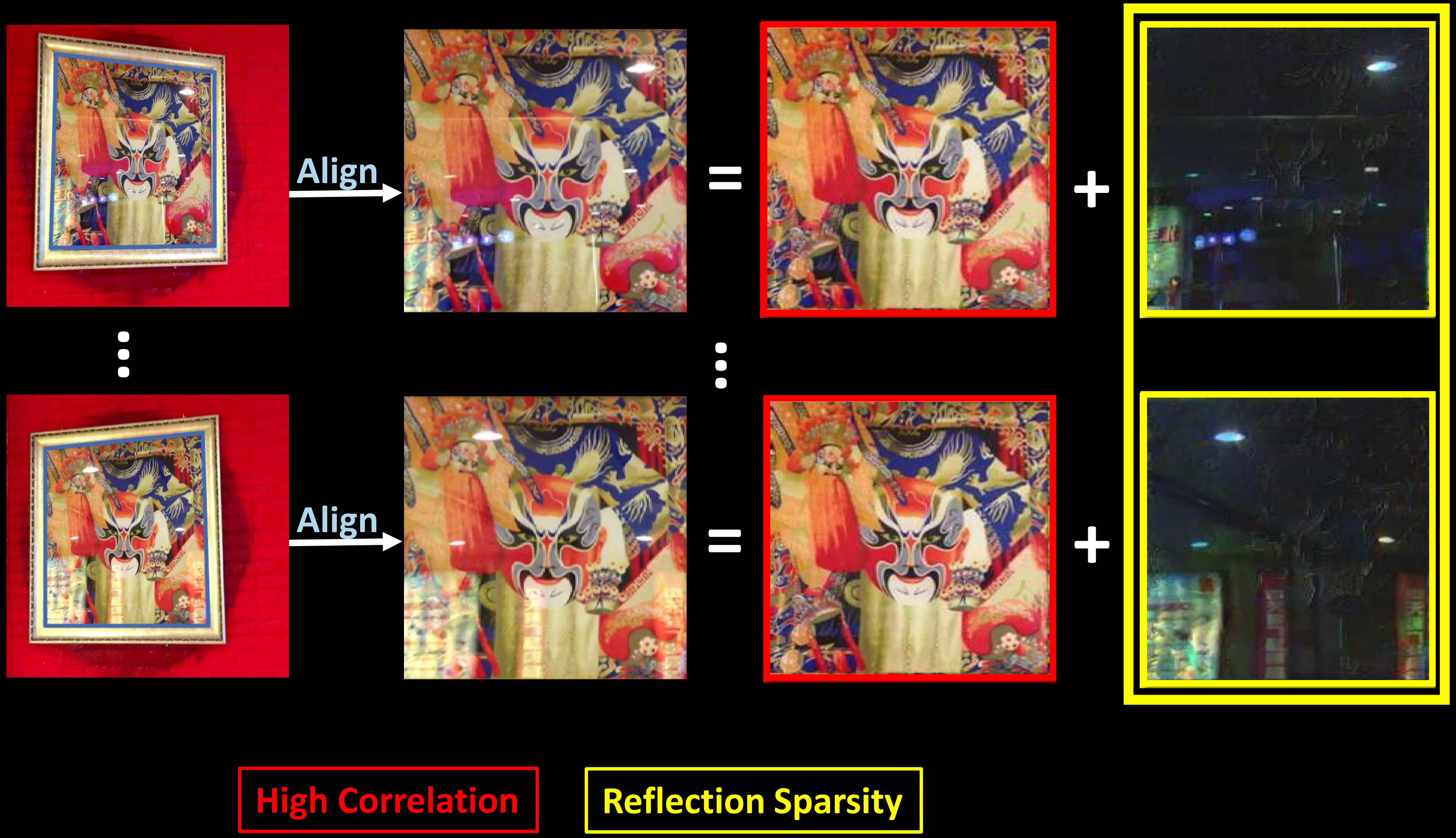


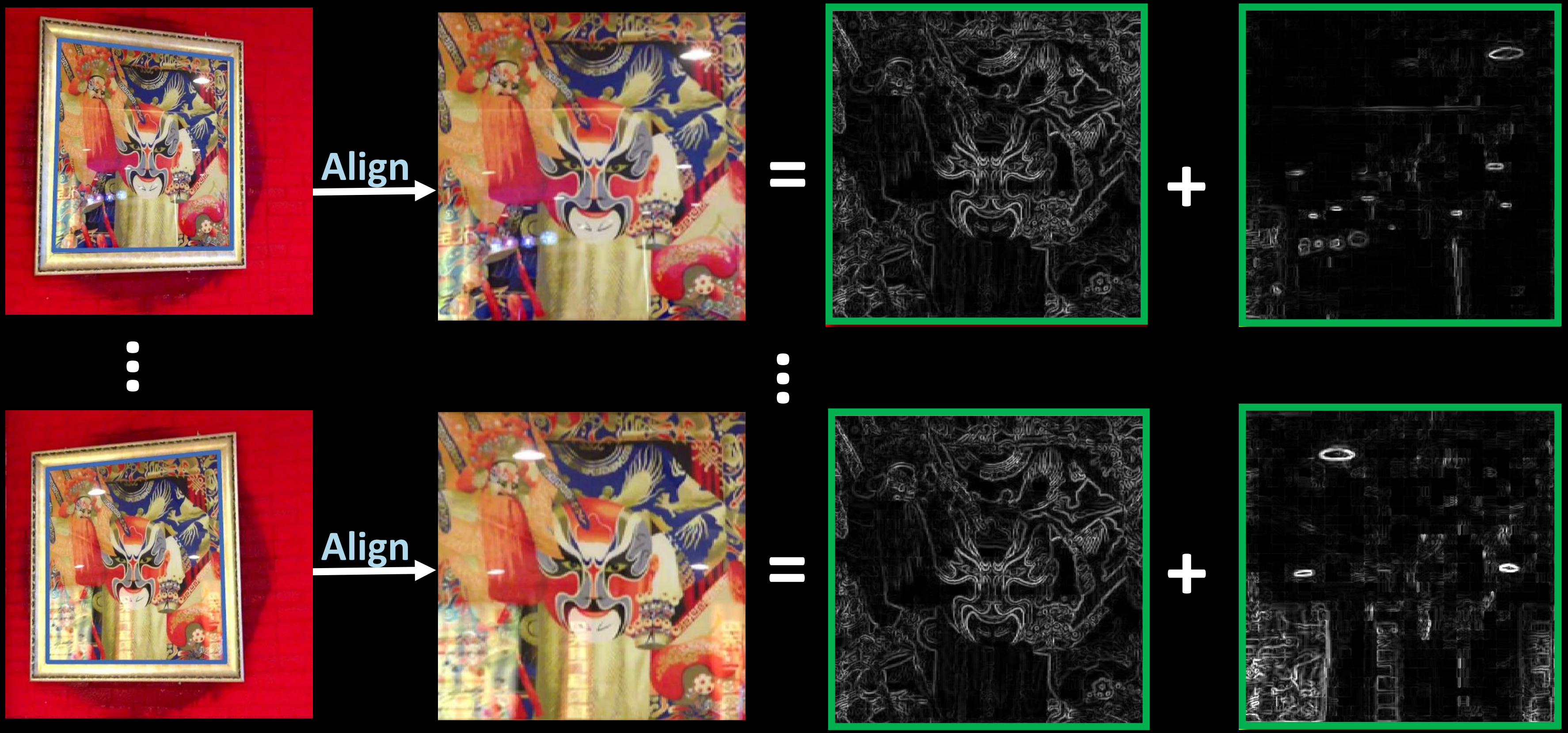
Observations & Priors



Observations & Priors



Observations & Priors



High Correlation

Reflection Sparsity

Gradient Sparsity

Observations & Priors

High Correlation | Reflection Sparsity | Gradient Sparsity | Gradient Independence

Problem Formulation

- The problem can be naturally formulated as:

$$\min \quad \mathrm{rank}(\boldsymbol{T}) + \lambda_1 \|\boldsymbol{M}\|_1 + \lambda_2 \|\boldsymbol{N}\|_F^2 + \lambda_3 \|\boldsymbol{DT}\|_0 + \lambda_4 \|\boldsymbol{DR}\|_0 + \lambda_5 \|\boldsymbol{DT} \odot \boldsymbol{DR}\|_0 + \lambda_6 \|\boldsymbol{\Omega} - \boldsymbol{DT} - \boldsymbol{DR}\|_F^2$$

$$\mathrm{s.t.} \quad \boldsymbol{F} \circ \Gamma = \boldsymbol{T} + \boldsymbol{M}; \boldsymbol{M} = \boldsymbol{R} + \boldsymbol{N}; \boldsymbol{T} \succeq 0; \quad \boldsymbol{R} \succeq 0;$$

Problem Formulation

- The problem can be naturally formulated as:

Reflection Sparsity

Correlation

Noise

Gradient Field Sparsity

$$\|\boldsymbol{DT}\|_0 \equiv \sum_{i=1}^{n}\sum_{j=1}^{2}\|\boldsymbol{d}_j * \boldsymbol{T}_i\|_0$$

Where * is convolution, d1 and d2 are horizontal and vertical derivative filters, respectively.

Gradient Field Independence

$$\boldsymbol{\Omega} \doteq \boldsymbol{DF}$$

$\odot$ is element wise product.

$$\min \ \mathrm{rank}(\boldsymbol{T}) + \lambda_1\|\boldsymbol{M}\|_1 + \lambda_2\|\boldsymbol{N}\|_F^2 + \lambda_3\|\boldsymbol{DT}\|_0 + \lambda_4\|\boldsymbol{DR}\|_0 + \lambda_5\|\boldsymbol{DT}\odot\boldsymbol{DR}\|_0 + \lambda_6\|\boldsymbol{\Omega} - \boldsymbol{DT} - \boldsymbol{DR}\|_F^2$$

$$\mathrm{s.t.}\ \boldsymbol{F}\circ\Gamma = \boldsymbol{T} + \boldsymbol{M};\ \boldsymbol{M} = \boldsymbol{R} + \boldsymbol{N};\ \boldsymbol{T}\succeq 0;\ \ \boldsymbol{R}\succeq 0;$$

By assuming the targeting region lies on a (nearly) planar surface, 2D homographs can be found to transform the misaligned regions to well aligned.

The values of both the transmitted and reflected layers are non-negative.

Convex Relaxation, Linearization and Optimization

Two main difficulties remain the problem intractable:

- The non-convexity of the Rank operator and the $\mathcal{L}^0$ norm.

  Replace the rank function and the $\mathcal{L}^0$ norm with the nuclear norm and $\mathcal{L}^1$ norm.

- The non-linearity of the constraint $\boldsymbol{F} \circ \Gamma = \boldsymbol{T} + \boldsymbol{M}$.

  Linearize the constraint by $\boldsymbol{F} \circ \Gamma + \sum_{i=1}^{n} \boldsymbol{J}_i \Delta\Gamma \epsilon_i \epsilon_i^T = \boldsymbol{T} + \boldsymbol{M}$.

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Final formulation of the separation problem is:

$$\min \|\boldsymbol{T}\|_* + \lambda_1 \|\boldsymbol{M}\|_1 + \lambda_2 \|\boldsymbol{N}\|_F^2 + \lambda_3 \|\boldsymbol{DT}\|_1 + \lambda_4 \|\boldsymbol{DR}\|_1 + \lambda_5 \|\boldsymbol{DT} \odot \boldsymbol{DR}\|_1 + \lambda_6 \|\boldsymbol{\Omega} - \boldsymbol{DT} - \boldsymbol{DR}\|_F^2,$$

$$\text{s.t.} \quad \boldsymbol{F} \circ \Gamma + \sum_{i=1}^{n} \boldsymbol{J}_i \Delta\Gamma \epsilon_i \epsilon_i^T = \boldsymbol{T} + \boldsymbol{M}; \;\; \boldsymbol{M} = \boldsymbol{R} + \boldsymbol{N}; \boldsymbol{T} \succeq 0; \;\; \boldsymbol{R} \succeq 0. \qquad (1)$$

Algorithm-Superimposed Image Decomposition

Algorithm 1: SID: Superimposed Image Decomposition

Input: $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0, \lambda_4 > 0, \lambda_5 > 0, \lambda_6 > 0$. The observation $\boldsymbol{F}$, and the initial transformation Γ.

while *not converged* **do**
- $\boldsymbol{L}^0 = \boldsymbol{M}^0 = \boldsymbol{N}^0 = \boldsymbol{T}^0 = \boldsymbol{R}^0 = \boldsymbol{Z}_1^0 = \boldsymbol{Z}_2^0 = \boldsymbol{Z}_3^0 = \boldsymbol{0} \in \mathbb{R}^{m\times n}$, $\Delta\boldsymbol{\Gamma}^t = \boldsymbol{0}, t = 0, \mu^0 > 0, \rho > 1$, $\boldsymbol{K}^0 = \boldsymbol{Q}^0 = \boldsymbol{Z}_4^0 = \boldsymbol{Z}_5^0 = \boldsymbol{0} \in \mathbb{R}^{2m\times n}$. Compute the warped areas $\boldsymbol{F} \circ \Gamma$ and their Jacobians $\boldsymbol{J}_i = \frac{\partial}{\partial \tau_i}\boldsymbol{F}_i \circ \tau_i$.
- **while** *not converged* **do**
  - Update $\boldsymbol{L}^{t+1}$
  - Update $\boldsymbol{M}^{t+1}$
  - Update $\boldsymbol{N}^{t+1}$
  - Update $\boldsymbol{K}^{t+1}$
  - Update $\boldsymbol{Q}^{t+1}$
  - **for** *i from* 1 *to n* **do**
    - Update $\boldsymbol{T}_i^{t+1}$
    - Update $\boldsymbol{R}_i^{t+1}$
  - **end**
  - $\boldsymbol{T}^{t+1}(\boldsymbol{T}^{t+1} < 0) = 0; \boldsymbol{R}^{t+1}(\boldsymbol{R}^{t+1} < 0) = 0;$
  - Update $\Delta\Gamma^{t+1}$
  - Update the multipliers
  - $\mu^{t+1} = \mu^t \rho; t = t + 1;$
- **end**
- $\Gamma = \Gamma + \Delta\Gamma^t;$

end

Output: Optimal solution $(\boldsymbol{T}^* = \boldsymbol{T}^t, \boldsymbol{R}^* = \boldsymbol{R}^t)$.

We empirically set $\lambda_1 = 0.3w, \lambda_2 = 50w, \lambda_3 = 1w, \lambda_4 = 5w$, $\lambda_5 = 50w$ and $\lambda_6 = 50w$ with $w = \frac{1}{\sqrt{m}}$. The initial transformation is obtained by feature matching with RANSAC

Inner loop iteratively solves the problem (1) with an updated transformation. The inner loop is stopped when

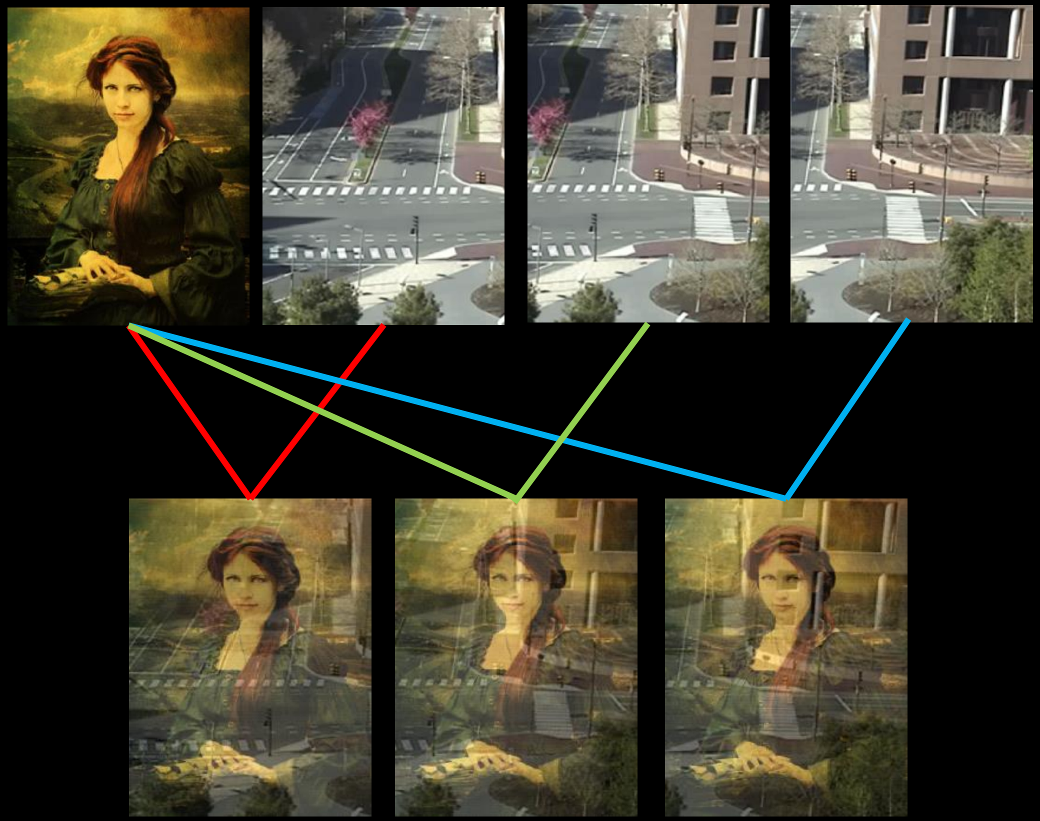
$$\|\boldsymbol{F} \circ \Gamma + \sum_{i=1}^{n} \boldsymbol{J}_i \Delta\Gamma^t \epsilon_i \epsilon_i^T - \boldsymbol{T}^{t+1} - \boldsymbol{M}^{t+1}\|_F \leq \delta \|\boldsymbol{F} \circ \Gamma\|_F$$

with $\delta = 10^{-6}$, or the maximal number of iteration is reached.

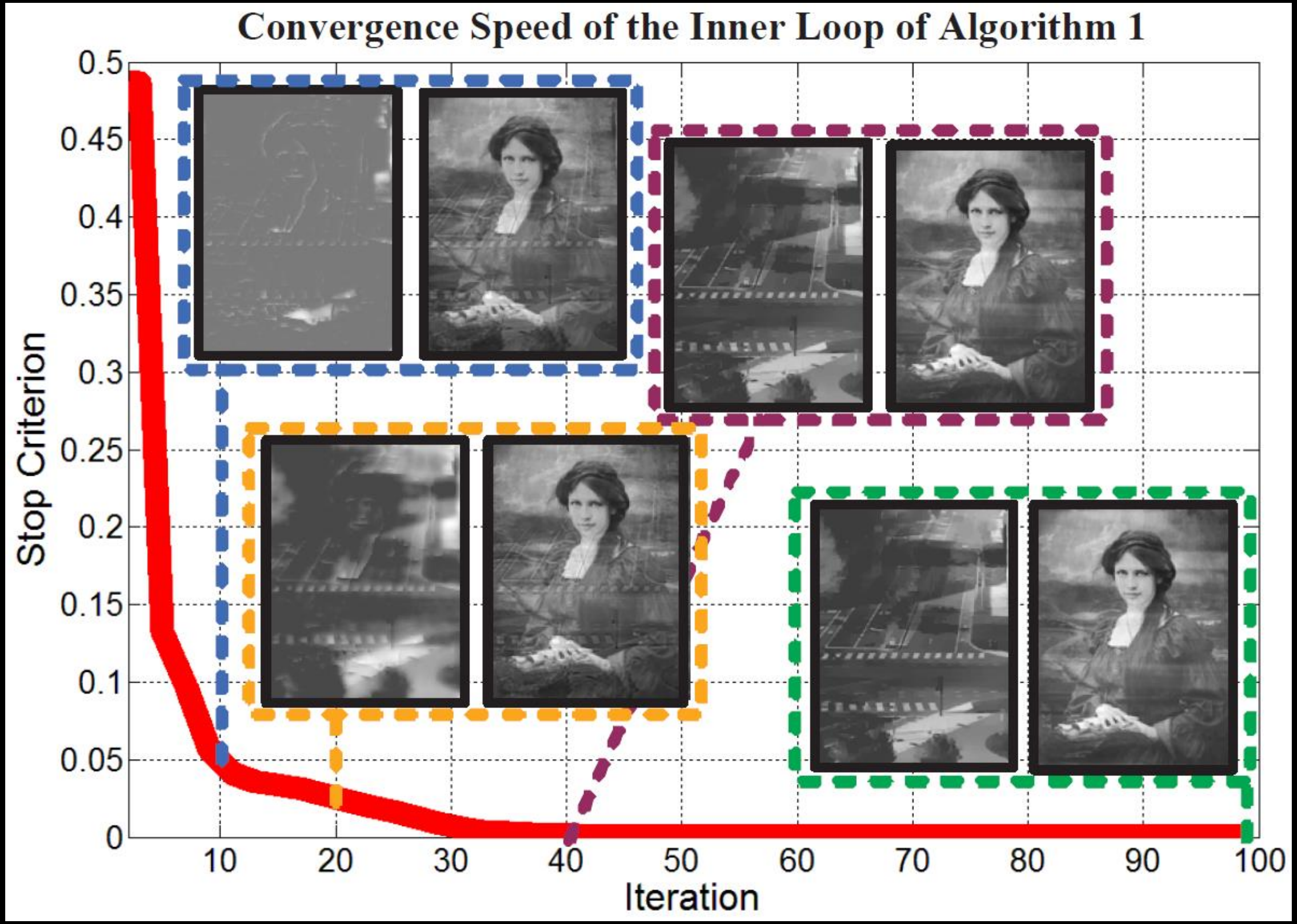
Outer loop is terminated when the change between two neighboring iterations is small enough, or the maximal number of iteration is reached

Simulation-Convergence Speed of Inner Loop

First: Transmitted Layer. Rest: Reflections.



3 of 15 synthesized superposed images.



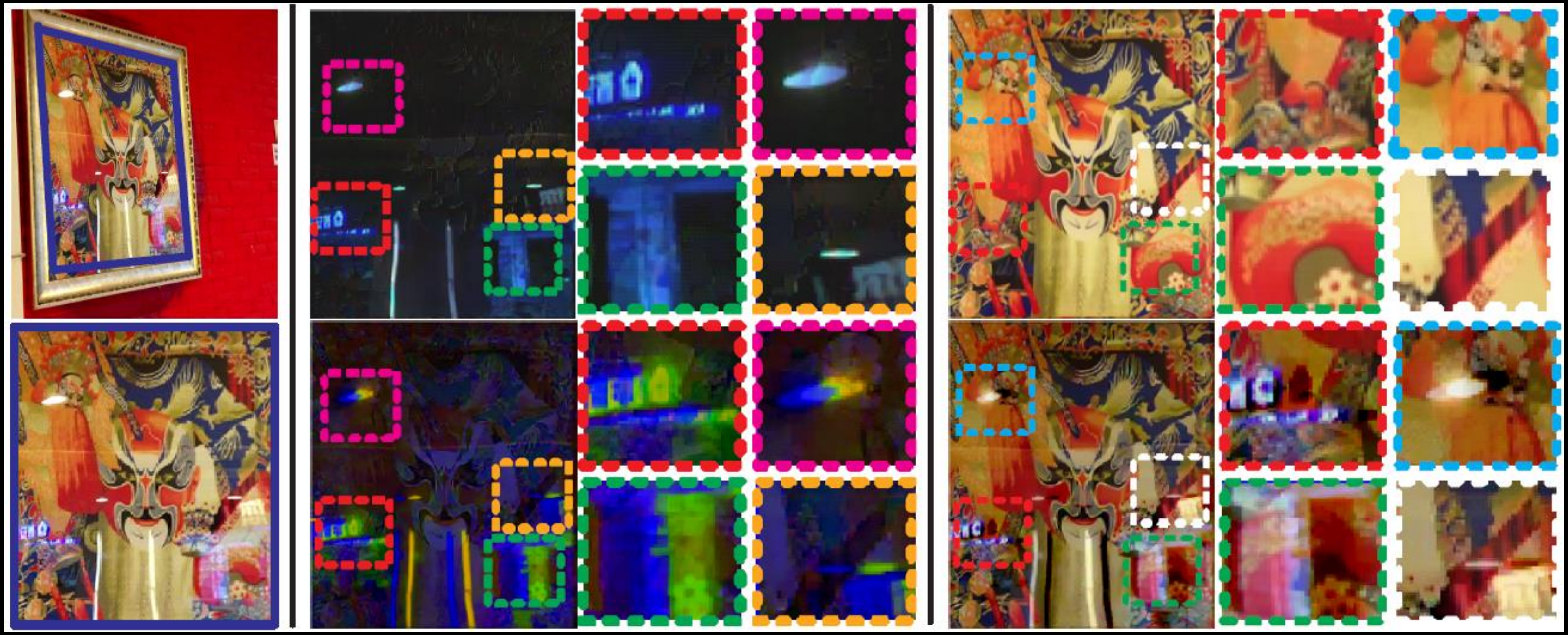
The result at the 40th iteration is very close to that at the 100th.

Experiment-Benefit of Alignment

The results after 1st iter. of outer loop

The results after 10st iter. of outer loop

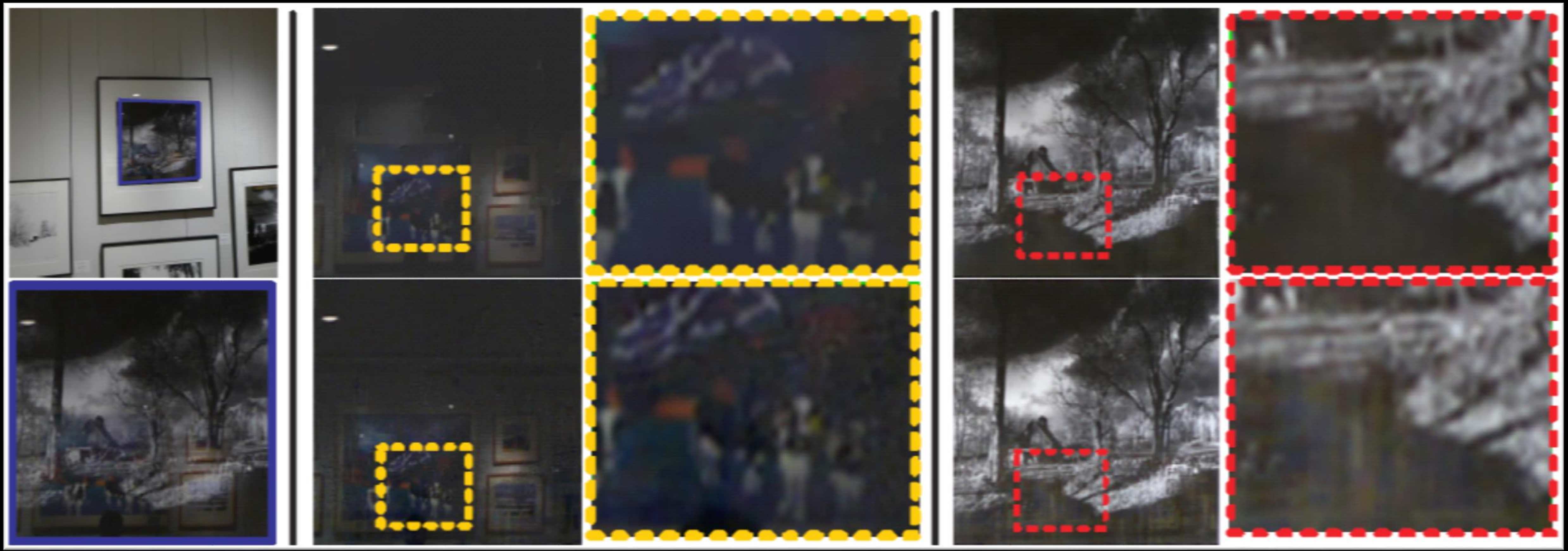
Experiment-Comparison on Real Data with SPBS-M[1]



Comparison with SPBS-M. The top row is from our method, while the bottom from SPBS-M [1].

[1] Blind separation of superimposed moving images using image statistics, Gai et al., TPAMI, 2012.

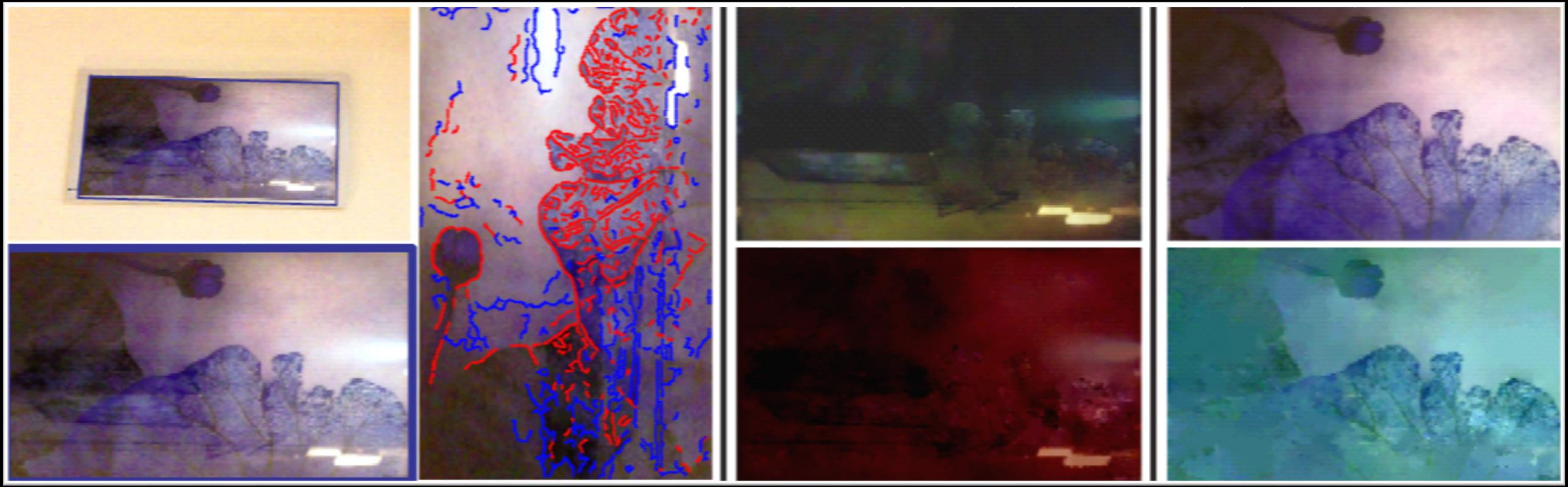
Experiment-Comparison on Real Data with RASL[2]



Comparison with RASL. The top row is from our method, while the bottom from RASL [2].

[2] RASL: Robust Alignment by Sparse and Low-Rank Decomposition for linearly correlated Images, Peng et al., TPAMI, 2012

Experiment-Comparison on Real Data with SIUA[3]



Comparison with SIUA. The top row is from our method, while the bottom from SIUA [3].

[3] User Asissted Separation of Reflections from a Single Image Using a Sparsity Prior, Levin and Weiss, TPAMI, 2007

Experiment-Failure Case

Original frame is blurred

Recovered reflection

Recovered transmitted layer

- Recovered reflection: largely correct + some ghosting effect.
- Recovered transmitted layer: blur reduced + good quality (correlation).

Conclusion

- **Several priors, including correlation, reflection-sparsity, gradient-sparsity and gradient-independence, have been exploited to make the problem well-defined.**
- **An efficient ALM-ADM based algorithm has been designed to seek the optimal solution.**
- **Both the two layers recovered by our method are with high qualities.**

Thank you for your attention!

Any Questions?

The code is available at:

cs.tju.edu.cn/orgs/vision/~xguo/homepage.htm

If any problem, please do not hesitate to contact Xiaojie Guo

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